Irreversible Investment, Capital Costs and Productivity Growth: Implications for Telecommunications

JEFFREY I. BERNSTEIN *

Department of Economics, Florida International University, Miami

THEOFANIS P. MAMUNEAS

Department of Economics, University of Cyprus

Abstract

This paper develops a model incorporating costly disinvestment and estimates the associated commitment premium required to invest in telecommunications. Results indicate that the irreversibility premium raises the opportunity cost of capital by 70 percent. This implies an average annual hurdle rate of return of 14 percent over the period 1986-2002. Irreversibility creates a distinction between observed and adjusted TFP growth. Observed growth, which omits the premium, annually averaged 2.8 percent from 1986 to 2002. This rate exceeded the (premium) adjusted TFP growth by 0.7 percentage points, therefore the average annual observed productivity growth overestimated the corrected rate by 33 percent.

1 Introduction

Investment in network industries such as telecommunications typically involves industry-specific capital so that investment reversibility is virtually impossible. Irreversible investment implies that a firm must incur substantial costs as it attempts to disinvest, and accordingly, capital cannot be shed like many other inputs. If network investments were reversible, a firm could readily disinvest when market conditions become unfavorable, thereby avoiding the financial consequences of these adverse conditions. However, because network investments are generally not fungible (in other words, they have limited alternative uses) a firm operating in a particular network industry commits to production in that industry. This commitment is costly. The reason is that a firm's ability to evolve through business conditions is relatively more constrained compared to firms undertaking fungible investment, while otherwise facing identical conditions. Therefore, as a consequence of the inability to disinvest, the "hurdle" rate of return on capital must exceed the opportunity cost pertaining to circumstances when disinvestment is viable. The first

* Contact author. Department of Economics, Florida International University, 11200 S.W. 8th Street, Miami, Florida 33199. Email: jeffrey.bernstein@fiu.edu The authors would like to thank an anonymous referee for helpful comments.

purpose of this paper is to develop a model of production and investment that incorporates costly disinvestment. The model is subsequently applied to telecommunications in order to estimate the magnitude of the commitment premium in that industry. In telecommunications, copper and fiber optic cables are typical examples of network infrastructure where reversing investment is prohibitively costly. Estimates of a non-zero premium provide evidence of the costs associated with irreversible investment.

Firms in the telecommunications industry referred to as incumbent local exchange carriers (ILECs) operate under an "obligation to serve" constraint, which requires capital to be on hand in order to stand ready to serve customers of basic landline telephone services.[1] This obligation represents an additional cost of investment in the context of irreversibility. If investment is reversible, ILECs could simply disinvest when basic landline telephone demand falls and reinvest when demand rises. An example of this behavior is found in the airline industry where aircraft are added or subtracted as required – indeed this is why planes are often referred to as "capital on wheels". But with irreversible investment, once capital is added to provide anticipated required capacity, it cannot be cost-effectively resold if the additional demand does not materialize.[2] Practically though, how likely is it for basic landline demand to fall and subsequently facility utilization decreases? After all, even if customers migrate to non-incumbent carriers who lease facilities from ILECs, these facilities still remain utilized as non-incumbents provide services to their own customers. But not all customers just migrate to other carriers; they migrate to alternative services from basic landline services. Wireless services, cable-based telephony and Internet-based services are substitutes for basic wireline services. Thus, as demands for these substitutes grow, all other things constant, competition in telecommunications markets reduces the basic landline customer base served by any one specific ILEC, as well as the customer base for all ILECs combined.[3]

A further complication is the problem of uncertain returns to investment.[4] Future market conditions, by their very nature, are uncertain and so network infrastructure investment will necessarily yield uncertain returns over its useful life. Under both conditions, namely irreversibility and uncertainty, there is an option value to waiting rather than investing. Intuitively, when a firm makes an irreversible investment, it gives up its option to wait to see how uncertainty is resolved.[5] If business conditions turn unfavorable, the firm is unable to disinvest if such events occur and would have to bear the financial consequences of the unfavorable conditions. It could not sell the capital and recover the undepreciated original value of its investment expenditure.[6] Thus, the uncertain future

[1] These carriers are presently Verizon, AT&T and Quest. Also, long distance services and ancillary services, such as call-waiting and caller-id, are not subject to the constraint.
[2] Although incumbents have an obligation to serve, investment incentives remain important because this obligation only applies to basic landline service. Carriers still have discretion over their investment decisions. Further, it has also been argued that regulatory regimes requiring the leasing of incumbents' facilities add an additional burden or constraint regarding investment decisions (see Kahn 2004, Hausman 2003; and Tardiff 2002).
[3] To extent that existing facilities can be used for non-basic landline telephone services, such as DSL internet services, the costs associated with the obligation to serve are mitigated as basic landline demand falls.
[4] As previously noted, even without uncertainty, returns to irreversible investment require a premium.
[5] Dixit and Pindyck (1994) provide a theoretical development of option values and investment.
[6] Economides (2002) has argued that returns to investment in telecommunications are quite stable. This however, is an empirical question that we address below in section 4 when alternative expectation processes are considered. Nevertheless, as noted above, even absent future uncertainty irreversibility adds to the costs of investment.

market value of network infrastructure requires a firm to form expectations regarding future prices of network facilities in order to formulate its investment plan. This paper incorporates uncertainty as future telecommunications capital acquisition prices are assumed to be random variables. Consistent with rational expectations, the parameters of the stochastic process are jointly estimated with the commitment premium associated with irreversible investment. Moreover, jointly estimating the expectations and commitment parameters improves the econometric efficiency of the estimates, as well as capturing the interrelationship between uncertainty and irreversibility.

An important indicator of dynamic performance is the efficiency by which inputs are transformed into outputs. This measure is referred to as the rate of total factor productivity *(*TFP*)* growth*.* TFP growth is calculated typically as the difference between a weighted average of output quantity growth rates (with revenue shares as weights) and a weighted average of input quantity growth rates (with cost shares as weights). In the context of irreversible investment, cost shares must include a possible commitment premium required to compensate firms for undertaking such investment. Since this commitment premium raises the opportunity cost of capital, it will affect the cost shares of the various inputs used in the production process, and accordingly, affect both input quantity growth and subsequent TFP growth rates. Productivity growth estimates for telecommunications generally exclude the costs of disinvestment. Therefore, "observed" TFP growth actually mismeasures the "correct or adjusted" rate, defined to include the appropriate measure of the opportunity cost of capital. The third purpose of this paper is to calculate both adjusted and observed TFP growth rates for telecommunication carriers and show how the two rates differ over time. An increase in the opportunity cost of capital due to the fact that investment is irreversible raises the capital cost share weight relative to the other input cost shares, and if capital is growing relatively faster (respectively slower) than the other factors of production, then observed input growth will understate (respectively overstate) the corrected rate of input growth. As a consequence observed, productivity growth will overstate (respectively understate) the appropriately adjusted rate of TFP growth.

This paper is organized in the following manner. Section 2 develops the model of investment which admits the possibility of costly disinvestment. This section shows that at the margin, these costs raise the capital input price (in other words the user cost of capital) relative to the case of reversible investment. Section 3 shows the calculation and decomposition of TFP growth. Both observed and adjusted TFP growth rates are considered in order to establish how the irreversibility premium affects the difference in productivity growth rates. Next, section 4 contains the empirical implementation and discussion of the findings. Results are presented on investment irreversibility margins, and differences in the rates of observed and adjusted productivity growth. Since the marginal cost of disinvestment affects the opportunity cost of capital, this section also provides a calculation of the hurdle rate of return to capital required to account for the costs of investment irreversibility. The last section of the paper provides a summary and conclusion.

2 Production and industry-specific investment

This section develops a model of production and investment when investment is irreversible or in other words, it is costly for the firm to disinvest. To begin, consider a transformation function written as:

$$F(y_t, v_t, t) = 0, \quad (1)$$

where y_t is an m dimensional vector of output quantities in period t, v_t is an n dimensional vector of input quantities in period t, and t also represents the exogenous disembodied technology index.[7]

Factor accumulation is represented by

$$v_{it} = x_{it} + (1-\delta_i)v_{it-1},\ i = 1,...,n, \quad (2)$$

where x_{it} is the addition to the i th input quantity in period t, and $0 \le \delta_i \le 1$ is the i th input depreciation rate. Since the depreciation rates for nondurable input quantities are defined as $\delta_i = 1$, in these cases from (2) $v_{it} = x_{it}$.

Input accumulation typified by facility investment in network industries is often viewed to be irreversible. For some types of investment, the physical characteristics associated with capital utilization make recouping expenditures through resale not financially viable. For example, in telecommunications, the majority of the cost of underground cable is the cost of burying the cable and not the cable itself, and so removing the cable for resale would be prohibitively expensive. For other types of investment (for example, switching equipment) that presumably could be "uninstalled" and resold, the problem is that the equipment is industry-specific and so its resale value is tied to the industrial business cycle. Thus, when conditions turn unfavorable and a firm attempts to disinvest, there are no buyers as concomitantly all firms want to sell such capital. This renders investment *de facto* irreversible.

Irreversible investment implies the cost of disinvestment must exceed the expected proceeds from the sale of network facilities. Further, when it is costly to reduce capital holdings, as a firm builds up infrastructure capital it becomes "harder" to reverse the increase. In other words, the capital cost per unit of capital exceeds the market price of the asset and this cost rises with the size of network facilities. The cost of irreversibility can be formalized by the function $I_i(v_{it})$, which is increasing in v_{it}, and at any future date it is always the case that $q_{it}^e v_{it} < I_i(v_{it})$ where q_{it}^e is the expected acquisition or purchase price of the ith capital in period t. From the later inequality, the proceeds obtained through the sale of infrastructure facilities do not compensate for the cost of disinvesting. As an example, the function $I_i(v_{it})$ represents the cost of extracting buried cable.[8] [9]

---

[7] The transformation function has the usual properties as described for example in Mas Collel, Whinston and Green (1995).

[8] The costs of irreversibility depend on the *level* of the capital stock. This differs from adjustment costs, which depend on the *change* in the capital stock. The reason for this difference is that if a firm decides not to invest in a particular period then adjustment costs are zero, but the costs of irreversibility do not disappear. As long as there are positive levels of network infrastructure there are costs to disinvesting. See Caballero (1999) for a survey on investment models.

Input demands are determined from minimizing the expected present value of acquisition and hiring costs. The uncertain future market value of network capital requires a firm to form expectations regarding these future prices in order to formulate its production and investment decisions. For example, in telecommunications, expectations involve the future prices of infrastructure capital such as switches, and copper cable.[10] Formally, the expected present value at time t (defined as the current time period) is given by the following:

$$\sum_{s=0}^{\infty}\sum_{i=1}^{n} a(t,t+s)[q^{e}_{it+s}x_{it+s} + I_i(v_{it+s})], \tag{3}$$

where q^{e}_{it+s} is the expectation in the current period t of the ith factor acquisition (or hiring) price in period $t+s$, and $a(t,t+s)$ is the discount factor with $a(t,t)=1$, $a(t,t+1)=(1+\rho_{t+1})^{-1}$, where ρ_{t+1} is the discount rate from period t to period $t+1$. The expression in (3) is minimized subject to equation sets (1), and (2). The Lagrangian for the problem is:

$$£ = \sum_{s=0}^{\infty} a(t,t+s)\left\{\sum_{i=1}^{n} q^{e}_{it+s}\left[v_{it+s} - (1-\delta_i)v_{it+s-1}\right] + I_i(v_{it+s}) - \lambda_{t+s}F\left(y^{e}_{t+s}, v_{t+s}, t+s\right)\right\} \tag{4}$$

where λ_{t+s} is the Lagrangian multiplier in period $t+s$. Differentiating (4) with respect to v_{it+s}, the first order condition for the i th input in period $t+s$ is:

$$a(t,t+s)\left[q^{e}_{it+s} + \frac{\partial I_i}{\partial v_{it+s}} - \lambda_{t+s}\frac{\partial F}{\partial v_{it+s}}\right] - a(t,t+s+1)(1-\delta_i)q^{e}_{it+s+1} = 0 \tag{5}$$

Dividing (5) by $a(t,t+s)$, and letting $a = a(t,t+s+1)/a(t,t+s)$ to be the constant discount factor, we have:

$$\lambda_{t+s}\frac{\partial F}{\partial v_{it+s}} = q^{e}_{it+s} + \frac{\partial I_i}{\partial v_{it+s}} - a(1-\delta_i)q^{e}_{it+s+1}, \tag{6}$$

and so:

$$\lambda_{t+s}\frac{\partial F}{\partial v_{it+s}} = \mu^{e}_{it+s}w^{e}_{it+s} = \omega^{e}_{it+s}, \tag{7}$$

---

[9] It is possible to assume that the cost of disinvesting for any one type of capital depends on all types. This formulation is not introduced because in the empirical implementation, there is one capital stock. In addition, the irreversibility cost function could be specified as $I_i(v_{it-1}, x_{it})$. Practically though, the marginal cost of disinvesting undepreciated facilities does not differ from the marginal cost associated with facility additions. Thus, it is assumed that $I_i(v_{it-1}, x_{it}) = I_i(v_{it-1} + x_{it}) = I_i(v_{it})$.

[10] Although companies enter into equipment contracts with suppliers, these contracts have specified termination dates.

where $w^e_{it+s} = q^e_{t+s} - aq^e_{it+s+1}(1-\delta_i)$, is the i th expected user cost in period $t+s$ when investment is reversible, and $\mu^e_{it+s} = \left(1+\frac{\partial I_i}{\partial v_{it+s}} / w^e_{it+s}\right)$, where $\mu^e_{it+s} \geq 1$, is the expected ith factor cost margin at period $t+s$. The term ω^e_{it+s} is the ith expected user cost in period $t+s$. User costs represent the opportunity cost of input utilization, and costly disinvestment causes the opportunity cost to increase at the margin by $\frac{\partial I_i}{\partial v_{it+s}} / w^e_{it+s}$ relative to the case of costless disinvestment. The term $\frac{\partial I_i}{\partial v_{it+s}} / w^e_{it+s}$ in μ^e_{it+s} is the (marginal) commitment premium required to undertake irreversible investment and since $\mu^e_{it+s} \geq 1$ (or $\frac{\partial I_i}{\partial v_{it+s}} \geq 0$), then a condition of irreversibility is $\mu^e_{it+s} > 1$. From expression (7), the value of the marginal product for the ith input equals its associated (expected) user cost, and with costly disinvestment, these marginal products exceed levels when investment is reversible. Next, having established how commitment premia affect the cost minimizing conditions at each point in time, the following section proceeds to identify the dynamic implications applicable to the measurement of productivity growth.

3 Costly disinvestment margins and TFP growth

The user cost derivation permits a recasting of the cost minimizing problem defined by (3) into the following equivalent form:

$$\min_{v_{it}} \sum_{i=1}^{n} \omega_{it} v_{it}, \tag{8}$$

subject to the transformation function given by (1), for periods $t = 0,...,\infty$. The problem in (8) relates to minimizing the production cost and leads to the first order conditions denoted by (7). From the equivalency of cost minimizing problems it is possible to define a cost function, which is denoted as:

$$C(\omega_{1t},..,\omega_{nt}, y_{1t},.., y_{mt}, t). \tag{9}$$

This function depends on user costs, therefore on the margins associated with costly disinvestment, expected acquisition and hiring prices, output quantities and the index of disembodied technology.

To calculate TFP growth, begin with the general cost function given by (9). Assuming that the cost function can be approximated by a function, with time-invariant second and higher order parameters, the cost difference between periods s and t, defined as $C_t - C_s$, will consist only of first order terms (see Denny and Fuss, 1983; and Bernstein, Mamuneas and Pashardes, 2004). Thus:

$$
\begin{aligned}
C_t - C_s &= .5\sum_{i=1}^{n}\left(\frac{\partial C}{\partial \omega_{it}} + \frac{\partial C}{\partial \omega_{is}}\right)(\omega_{it} - \omega_{is}) \\
&+ .5\sum_{j=1}^{m}\left(\frac{\partial C}{\partial y_{jt}} + \frac{\partial C}{\partial y_{js}}\right)(y_{jt} - y_{js}) \\
&+ .5\left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial s}\right)(t - s).
\end{aligned} \tag{10}
$$

Now TFP growth between periods, s and t embodying the margins associated with costly disinvestment is defined as $TFPG_n^a(s,t) = \dot{Y}/Y - \dot{\Psi}/\Psi$, where

$$
(\dot{Y}/Y) = \sum_{j=1}^{m} .5\left[\sigma_{jt}\,\frac{(y_{jt} - y_{js})}{y_{jm}}\,\frac{y_{jm}R_t}{R_m y_{jt}} + \sigma_{js}\,\frac{(y_{jt} - y_{js})}{y_{jm}}\,\frac{y_{jm}R_s}{R_m y_{js}}\right]
$$

is the output quantity growth rate and

$$
(\dot{\Psi}/\Psi) = \sum_{i=1}^{n} .5\left[\breve{s}_{it}\,\frac{(v_{it} - v_{is})}{v_{im}}\,\frac{v_{im}R_t}{R_m v_{it}} + \breve{s}_{is}\,\frac{(v_{it} - v_{is})}{v_{im}}\,\frac{v_{im}R_s}{R_m v_{is}}\right]
$$

is the adjusted input quantity growth rate, where p_j is the jth output price, $R_t = \sum_j p_{jt} y_{jt}$ is the total revenue, $\sigma_{jt} = p_{jt} y_{jt} / R_t$, is the $j$the output revenue share, and the ith cost share is defined as $\breve{s}_{it} = \omega_{it} v_{it} / R_t$. The cost shares are defined in terms of revenue because without loss of generality, it is possible to define an artificial input such that this $n+1$st input's price is $\omega_{n+1t} = R_t - C_t$, and its quantity is $v_{n+1t} = 1$, whereby $R_t. = \sum_{i=1}^{n+1} \omega_{it} v_{it}$. Moreover, $(v_{n+1t} - v_{n+1s}) = 0$, so the calculation of the input growth rate above does not involve the $n+1$ st input. The numerators of the cost shares involve the user costs, given as ω_{it}, so the shares include the marginal commitment premia associated with disinvestment. The subscript m signifies the average value so for example for output quantity, $y_m = .5(y_t + y_s)$.

From the Appendix, adjusted *TFP* growth equals:

$$
TFPG_n^a(s,t) = .5\left(\breve{\xi}_{\nu t} R_t + \breve{\xi}_{\nu s} R_s\right)\frac{1}{R_m}(t - s). \tag{11}
$$

Expression (11) shows that adjusted productivity growth represents technological change, where $\breve{\xi}_{\nu t}$ is the input-based rate of technological change in period t.

The difficulty calculating adjusted TFP growth is the unobservability of the marginal disinvestment cost and as a consequence, adjusted TFP growth is unobservable. Thus, measures of productivity growth typically use observed cost shares to compute the input growth rate. These observed cost shares by their very nature must exclude the unobserved marginal costs of disinvesting, so "observed" productivity growth is defined as $TFPG^o(s,t) = \dot{Y}/Y - \dot{V}/V$ where

$$(\dot{V}/V) = \sum_{i=1}^{n} .5\left[s_{it} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im}c_t}{c_m v_{it}} + s_{is} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im}c_s}{c_m v_{is}} \right]$$

is the observed input growth rate, $c_t = \sum_{i=1}^{n} w_{it}v_{it}$ is the observed total cost and $s_{it} = w_{it}v_{it} / c_t$ is the observed ith input cost share.

By applying the definition of adjusted and observed TFP growth rates, their relationship is provided by:

$$\begin{aligned} TFPG^o(s,t) = TFPG_n^a(s,t) & \\ & + \sum_{i=1}^{n} .5\left[\breve{s}_{it} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im}R_t}{R_m v_{it}} + \breve{s}_{is} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im}R_s}{R_m v_{is}} \right] \\ & - \sum_{i=1}^{n} .5\left[s_{it} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im}c_t}{c_m v_{it}} + s_{is} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im}c_s}{c_m v_{is}} \right]. \end{aligned} \tag{12}$$

Collecting terms in (12) and recall that $\breve{s}_{it} = \frac{\omega_{it}v_{it}}{R_t}$, $s_{it} = \frac{w_{it}v_{it}}{c_t}$ and $\omega_{it} = \mu_{it}w_{it}$ then:

$$\begin{aligned} TFPG^o(s,t) = TFPG_n^a(s,t) & \\ & + \sum_{i=1}^{n} .5\left(\breve{s}_{it} \frac{R_t}{v_{it}} + \breve{s}_{is} \frac{R_s}{v_{is}} \right) \frac{v_{im}}{R_m} \frac{(v_{it} - v_{is})}{v_{im}} \\ & - \sum_{i=1}^{n} .5\left(s_{it} \frac{c_t}{v_{it}} + s_{is} \frac{c_s}{v_{is}} \right) \frac{v_{im}}{c_m} \frac{(v_{it} - v_{is})}{v_{im}} \\ = TFPG_n^a(s,t) & \\ & + \sum_{i=1}^{n} .5\left(\mu_{it}w_{it} + \mu_{is}w_{is} \right) \frac{v_{im}}{R_m} \frac{(v_{it} - v_{is})}{v_{im}} \\ & - \sum_{i=1}^{n} .5\left(w_{it} + w_{is} \right) \frac{v_{im}}{c_m} \frac{(v_{it} - v_{is})}{v_{im}} \end{aligned}$$

(13)

Further using (11) yields:

$$
\begin{aligned}
TFPG^{o}(s,t) = {} & TFPG_{n}^{a}(s,t) \\
& + \sum_{i=1}^{n} .5\left(\mu_{it} w_{it} \frac{v_{im}}{R_m} - w_{it} \frac{v_{im}}{c_m} \right) \frac{(v_{it} - v_{is})}{v_{im}} \\
& + \sum_{i=1}^{n} .5\left(\mu_{is} w_{is} \frac{v_{im}}{R_m} - w_{is} \frac{v_{im}}{c_m} \right) \frac{(v_{it} - v_{is})}{v_{im}} \\
= {} & .5\left(\breve{\xi}_{vt} R_t + \breve{\xi}_{vs} R_s \right) \frac{1}{R_m} (t-s) \\
& + .5 \sum_{i=1}^{n} . \left[\left(\frac{\mu_{it}}{(1+M_m)} - 1 \right) \frac{w_{it} v_{im}}{c_m} + \left(\frac{\mu_{is}}{(1+M_m)} - 1 \right) \frac{w_{is} v_{im}}{c_m} \right] \frac{(v_{it} - v_{is})}{v_{im}}
\end{aligned}
\tag{14}
$$

where the second term on the right side of (14) reflects the marginal premia associated with irreversible investments relative to the observed gross profit margin, which is $R_m / c_m = 1 + M_m$, where $(R_m - c_m)/c_m = M_m$ is the observed profit margin.

Measures of observed TFP growth relying on growth accounting methods, as opposed to econometric methods based on the estimates of production or cost functions, necessarily assume total revenue equals total (observed) cost.[11] To be consistent with growth accounting methods, introduce an $n+1$ input such that the price of this input is total revenue minus observed cost or $R_t - c_t$, and its quantity is unity. Label this observed TFP growth inclusive of the accounting convention $TFPG_n^o(s,t)$ so that

$$
(\dot{V}/V) = \sum_{i=1}^{n+1} .5\left[s_{it} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im} R_t}{R_m v_{it}} + s_{is} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im} R_s}{R_m v_{is}} \right]
$$

is the new observed input growth rate such that the ith cost share is defined as $s_{it} = w_{it} v_{it} / R_t$. With this convention (14) becomes:

$$
\begin{aligned}
TFPG_n^o(s,t) = {} & .5\left(\breve{\xi}_{vt} R_t + \breve{\xi}_{vs} R_s \right) \frac{1}{R_m} (t-s) \\
& + .5 \sum_{i=1}^{n} . \left[(\mu_{it} - 1) \frac{w_{it} v_{im}}{R_m} + (\mu_{is} - 1) \frac{w_{is} v_{im}}{R_m} \right] \frac{(v_{it} - v_{is})}{v_{im}} .
\end{aligned}
\tag{15}
$$

Notice for the $n+1$st input $(v_{it} - v_{is}) = 0$, because quantity is always unity. Expression (15) shows that observed TFP growth consists of two terms. The first term is the rate of

[11] See Bernstein and Zarkadis (2004), Gollup (2000); and the FCC (1997).

technological change and the second term reflects the margins associated with the premia required by irreversible investments. Indeed, for positive input growth rates ($\frac{(v_{it}-v_{is})}{v_{im}} > 0$) when commitment premia cause adjusted input growth to exceed observed growth, then observed productivity growth ($TFPG^o$) overstates the correct or adjusted growth rate ($TFPG^a$). Notice if investment is reversible ($\mu_{it}=1$), then the difference between observed and adjusted TFP growth rates disappear and technological change accounts for productivity growth.

4 Empirical implementation and results

This section of the paper contains the estimation results, and in particular presents estimates of the margins associated costly disinvestment. In order to estimate the marginal cost of irreversibility, assume μ_{it} is time invariant so that $\omega_{it} = \mu_i w_{it}$, and thus equation (15) becomes:

$$TFPG_n^o(s,t) = \xi_t + \sum_{i=1}^{n} (\mu_i - 1) s_{im} \frac{(v_{it}-v_{is})}{v_{im}} \tag{16}$$

where $s_{im} = w_{im} v_{im} / R_m$ and $\xi_t = .5\left(\tilde{\xi}_{vt} R_t + \tilde{\xi}_{vs} R_s\right)\frac{1}{R_m}(t-s)$. Two cases are considered for the rate of technological change. The first case assumes the rate is constant so $\xi_t = \xi$.[12] The second case assumes the rate itself changes over time, so $\xi_t = \xi + \xi_T t$, therefore a time trend, t, is introduced into the estimating equation.[13]

Equation (16), which is the equation to be estimated, relates observed TFP growth to the rate of technological change, ξ_t, and input growth rates whose parameters, μ_i, are the margins arising from costly disinvestment. Data for the telecommunications industry are updated from Bernstein and Zarkadas (2004). Output growth rates are defined as the growth rates of a chained Fisher quantity index of local services, intrastate services and interstate services. Interstate output consists of the aggregation of access lines, interstate switched access minutes and special access lines. Access lines, measured by the sum of the number of business, public and residential access lines, and special access lines provide connectivity to the network, while the volume of interstate activity is measured by interstate switched access minutes. Local output is measured by the number of local calls. Intrastate output consists of the aggregation of intrastate toll minutes and intrastate switched access minutes.

There are three input categories, labor, capital and intermediate inputs. Labor quantity is based on annual data for the number of employees. The quantity of intermediate inputs is calculated as intermediate input expenses divided by a price index. Intermediate input

[12] Thus, $\xi_t = .5\left(\tilde{\xi}_{vt} R_t + \tilde{\xi}_{vs} R_s\right)\frac{1}{R_m}(t-s) = \tilde{\xi}_{vt}(.5(R_t + R_s)\frac{1}{R_m} = \tilde{\xi}_{vt} = \tilde{\xi}_{vs} = \xi$

[13] Using time dummies rather than a time trend did not materially affect the regression results.

expense is computed by subtracting from total operating expenses the sum of labor compensation and depreciation and amortization expense. The intermediate input price index is taken to be the gross domestic product price index. The capital input is the accumulation of constant dollar annual investment plus the depreciated value of the previous year's capital stock. The acquisition price of capital before income taxes is defined as $q_K = Q_K(1-\vartheta_K - \iota_K)/(1-u_c)$, where Q_K is a composite asset price index obtained from the Bureau of Economic Analysis (BEA), $\vartheta_K = u_c z_K$, where z_K is the present value of physical capital cost allowances, u_c is the corporate income tax rate and ι_K is the physical investment tax credit rate obtained from Xanthopoulos (1991) and the BEA. The discount rate is set to 4 percent, and the depreciation rate is 7.2 percent. The sample covers the period from 1985 to 2003.[14]

Since there is one capital input then the margins associated with irreversible investment for labor and intermediate inputs are $\mu_L = \mu_M = 1$, and from (16):

$$TFPG_n^o(s,t) = \xi_t + \left(\mu_K - 1\right) s_{Kt} \hat{v}_{Kt} \tag{17}$$

where $(\hat{\ })$ denotes the growth rate.

In addition, future values of the capital acquisition price are uncertain and this random variable is assumed to follow a first order autoregressive process of the following form

$$q_{Kt+1} = \phi_K + \theta_K q_{Kt} + e_{Kt} \tag{18}$$

where ϕ_K , and θ_K are parameters, the error e_{Kt} is identically and independently distributed over time, and since expectations are rational, the expected value of e_{Kt} is zero. Equations (17) and (18) are jointly estimated because the expected value of q_{Kt+1} forms part of the user cost of capital (from equation (6)), which in turn enters into the capital cost share in equation (17).[15]

Table 1 reports the parameter estimates. This table shows the Least Squares estimates for static (that is $q_{Kt+1} = q_{Kt}$) and autoregressive expectations for the capital acquisition price, and also provides Instrumental Variable (IV) estimates. The set of instruments consists of a constant, the twice lagged values of the input and output growth rates, the twice lagged value of the acquisition price of capital and the lagged values of the cost shares. The estimates for each of these three cases are also reported for constant and time varying rates of technological change. First from the standard error of the regression (SER) statistic, the results show that in each case the time varying rate of technological change outperforms the constant rate version. Second, although the parameter estimates are quite similar across models with time varying rate of technological change, the autoregressive expectations model is preferred to the other two. Indeed, testing the null hypothesis that

[14] For a detail description and sources of data ,see Bernstein and Zarkadas (2004).
[15] A second order process was also estimated but the first order case could not be rejected.

expectations are static, in other words $\phi_K = 0$ and $\theta_K = 1$, a Wald test shows that the null hypothesis is rejected as the value of the test statistic is 6.59, which exceeds the critical value $\chi^2_{2,0.05} =$ 5.99.

Next, to see if the commitment premium required to undertake irreversible investment is nonzero (or that $\mu_K > 1$), the model has been reestimated with $(\mu_K - 1)$ replaced with a parameter α to test the null hypothesis that $\alpha = 0$. The test is based on the preferred model, which is the autoregressive expectations version with time varying rate of technological change. The hypothesis that $\mu_K = 1$ is rejected with a $t-statisic$ equal to 2.49.[16] Therefore, the estimation results indicate that investment is irreversible with $\mu_K = 1.7$ (from the preferred model, which is the fourth column of estimation results). This finding implies that the associated commitment premium increases the user cost of capital by 70%, and as shown in table 2 annually averages to be 0.270. Since the margin between irreversible and reversible investment is a constant (as $\mu_K = 1.7$) the user costs inclusive of the premium are readily converted to those without the premium by dividing the former by 1.7).

It is also possible to restate the commitment premium in terms of the cost of capital or equivalently the hurdle rate of return. Recall from (7) that $\omega_{Kt} = \mu_K w_{Kt} = \mu_K \left[q_{Kt} - a q^e_{Kt+1}(1-\delta) \right]$, where $a = 1/(1+\rho)$ is the discount factor and ρ is the rate of return. Let $\gamma = 1/(1+\rho^*)$, such that ρ^* is the hurdle rate of return, which makes the user costs of capital inclusive of the premium, ω_{Kt}, equal to the user cost exclusive of the premium. Thus:

$$\omega_{Kt} = q_{Kt} - \gamma q^e_{Kt+1}(1-\delta)$$

$$\gamma = \frac{q_{Kt} - \omega_{Kt}}{q^e_{Kt+1}(1-\delta)}$$

therefore $\rho^* = \frac{1}{\gamma} - 1$.[17] The expression $(\rho^* - \rho)/\rho$ shows the bias in the rate of return on capital attributable to the exclusion of the marginal cost associated with the inability to disinvest. Table 2 reports the estimates of ρ^*. Further, to discern the effect on the hurdle rate of return of inappropriately assuming static expectations, this table also presents the calculation of ρ^* under the assumption of static expectations. First, our findings indicate on average the annual hurdle rate inclusive of the commitment premium is 2.5 times greater than the assumed rate excluding the premium (with $\rho^* = 0.14$ and $\rho = 0.04$ then

[16]Also based on the preferred model, the hypothesis that the time trend has no effect ($\xi_t = 0$) is rejected with a $t-statistic$ equal to -3.31. In addition, the test that $\xi_t = 0$ and $\mu_K = 1$ is rejected since the Wald test statistic is equal to 9.77, higher than the critical value of $\chi^2_{2,0.05} =$ 5.99.

[17] Under static expectations the formula for γ simplifies to $\gamma = \frac{1-\mu_K[1-\alpha(1-\delta_K)]}{(1-\delta_K)}$

$2.5 = (0.14 - 0.04)/0.04$). Therefore, omitting the commitment premium arising from irreversible investment underestimates the appropriate rate of return on capital. Second, with static expectations the average hurdle rate is estimated to be $\rho^* = 0.129$. This rate is 8.5 percent below the rate based on rational price expectations. But differences in average hurdle rates obfuscate an important outcome. As table 2 shows, the sample standard deviation of ρ^* under static expectations is much higher than the standard deviation associated with rational expectations. This implies considerable sample instability with static expectations, and indeed for many years, hurdle rate estimates derived by mistakenly assuming static expectations are substantially less than the rates obtained from the rational expectations model. There is little empirical research to compare these findings. However, a paper by Pindyck (2005) using different methods and framework concludes that the telecommunications hurdle rate with irreversible investment is between 14.2 and 17.5 percent. This result is very similar to the estimates obtained in this paper.

Table 3 reports the TFP growth rates and their decomposition. First, the generally positive rate of technological change contributes to productivity growth and over the period 1986-2002 the average annual rate of technological change was about 1.7 percent. Second, the observed TFP growth rate normalized such that total revenue equals total observed cost ($TFPG_n^o$) is generally positive and varies over the sample period. The average annual rate over the years from 1986 to 2002 was 2.8 percent. The preceding concept of productivity growth uses growth accounting methods as do many earlier studies of telecommunications' productivity. For example, Bernstein and Zarkadas (2004) found that over the period 1986-2001 productivity growth averaged either 2.7 percent or 2.9 percent depending on the disaggregation of intrastate outputs. The results in this paper are comparable and provide growth rates for the most current time period.

For the first time, this paper introduces estimates of the premium due to irreversible investment into the measurement of TFP growth. Since the commitment premium exceeds unity and with growing capital, this premium reduces adjusted productivity growth relative to the observed rate (that is, from equation (17), $(\mu_K - 1) s_K \acute{v}_K > 0$). Therefore, observed TFP growth overestimates adjusted productivity growth. Table 3 shows this result as observed productivity growth, $TFPG^o$, consistently exceeds the appropriately adjusted rate, $TFPG^a$. Indeed, on average, over the sample period observed, productivity growth annually overestimates adjusted growth by 0.7 percentage points per year. This bias is relatively large and represents a 33 percent productivity overvaluation when the costs of irreversible investment are incorrectly omitted from the analysis.

5 Summary and conclusion

A major purpose of this paper was to develop a model incorporating costly disinvestment in order to estimate the premium on the opportunity cost of capital in telecommunications where investment is often perceived as irreversible. Irreversible investment, for example in telecommunications copper and fibre optic cables, implies that a firm must incur substantial costs as it attempts to dispose of its capital stocks. As a consequence, firms commit to production. This is a costly commitment whereby the inability to disinvest raises the hurdle rate of return on capital above the opportunity cost associated with reversible investment. The findings in this paper indicate that telecommunications investment is indeed irreversible, and the opportunity cost of capital is 70 percent greater than would be the case under reversible investment. In terms of the hurdle rate of return on capital, costly disinvestment leads to an average annual rate of 14 percent, which is two and a half times the assumed return under reversible investment.

Because of the commitment premium, this paper distinguishes between observed and adjusted TFP growth rates. The observed rate, which is the rate typically calculated in productivity studies, omits the premium. The premium raises the opportunity cost of capital and thereby affects the input cost shares used to compute input quantity growth. As a consequence, the revised input growth leads to the calculation of adjusted TFP growth rates. This paper finds calculates the average annual rate of observed productivity growth over the years from 1986 to 2002 to be 2.8 percent. On average, this rate overestimates adjusted growth by 0.7 percentage points per year. Indeed, this bias is quite large, representing an annual 33 percent productivity overvaluation when the significant costs of irreversible investment are inappropriately excluded.

There are a number of avenues for future research. Probably the most important direction is to translate the capital commitment premium into marginal costs of production for the various telecommunication services. With these marginal costs, it would then be possible to discern the proper basis to set regulated telecommunications prices, notably wholesale prices for access to the telecommunications network. If telecommunication carriers provided a single service, or if they produced multiple services in a fixed proportion, or priced these services according to a common markup (or markups in a fixed proportion to each other), the translation from commitment premium to marginal cost would be relatively straightforward. However, in the context of multiple services and distinct marginal costs, the translation from input premium to marginal costs of production requires knowledge of the multiple-product cost function. Estimation of this function in the context of costly reversible investment is the next step in our research program.

6 References

Bernstein, J. I., T. P. Mamuneas and P. Pashardes (2004) “Technical Efficiency and U.S. Manufacturing Productivity Growth,” *The Review of Economics and Statistics*, 61: 402-412.

Bernstein, J. I. and C. J. Zarkadas (2004) “Measurement of TFP growth for US Telecommunications,” in R. Cooper and G. Madden (ed.), *Frontiers of Broadband, Electronic and Mobile Commerce*, Physica-Verlag: Heidelberg.

Caballero, Ricardo J. (1999) “Aggregate Investment,” in John B. Taylor and Michael Woodford (ed.), *Handbook of Macroeconomics*, Amsterdam Elsevier: Amsterdam: 813-862.

Denny, M. and M. Fuss (1983) “The Use of Discrete Variables in Superlative Index Number Comparisons,” *International Economic Review*, 24: 419-421.

Dixit, Avinash K. and Robert S. Pindyck (1994) *Investment Under Uncertainty*. Princeton University Press: Princeton

Economides, Nicholas (2002) "Real Options and the Costs of the Local Telecommunications Network," in James Alleman and Eli Noam (ed.), *The New Investment Theory of Real Options and Its Implications for Telecommunications Economics*, Kluwer Academic Publishers: Boston: 207-213.

FCC (1997) Price Cap Performance Review for Local Exchange Carriers, Fourth Report and Order in *CC Docket No. 94-1 and Second Report and Order in CC Docket No. 96-262.*

Gollop, Frank M. (2000) The FCC X-Factor: 1996-1998 Update, Attachment 4 USTA Comments in *CC Docket No. 94-1*, *96-262*.

Hausman, Jerry A. (2003) "Regulated Costs and Prices in Telecommunications," in G. Madden (ed.), *Emerging Telecommunications Networks*, Edward Elgar Publishing: Cheltenham.

Kahn, Alfred E. (2004) *Lessons from Deregulation*, AEI-Brookings Joint Center for Regulatory Studies, Washington, D.C.

Mas-Collel, A., M. Whinston, and J. Green (1995) *Microeconomic Theory*. Oxford University Press: New York.

Pindyck, Robert S. (1988) "Irreversible Investment, Capacity Choice, and the Value of the Firm," *American Economic Review*, 79: 969-985.

Pindyck, Robert S. (2005) Pricing Capital Under Mandatory Unbundling and Facilities Sharing, *NBER Working Paper No.* 11225.

Tardiff, Timothy J. (2002) "The Forecasting Implications of Telecommunications Cost Models," in James Alleman and Eli Noam (ed.), *The New Investment Theory of Real Options and Its Implication For Telecommunications Economics*, Kluwer Academic Publishers: Boston: 181-190.

Xanthopoulos, J. A. (1991) *Cost of Capital Model of Effective Tax Rates: Forty Years of Tax Policy and the Effects on Marginal Effective Tax Rates*, Ph.D. Thesis, University of Maryland.

7 Appendix

From equation (10), applying Shephard's lemma $\frac{\partial C}{\partial \omega_{it}} = v_{it}$, and noting from (8) that $C_t - C_s = \sum \omega_{it} v_{it} - \sum \omega_{is} v_{is}$, then after collecting terms (10) becomes:

$$\sum_{i=1}^{n} .5(\omega_{it} + \omega_{is})(v_{it} - v_{is}) = \sum_{j=1}^{m} .5\left(\frac{\partial C}{\partial y_{jt}} + \frac{\partial C}{\partial y_{js}}\right)(y_{jt} - y_{js}) + .5\left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial s}\right)(t - s) \tag{A1}$$

Multiplying (A1) by -1 and adding $\sum_j .5\left(p_{jt} + p_{js}\right)\frac{C_m}{R_m}(y_{jt} - y_{js})$ to both sides, where p_j is the jth output price, $C_m = .5(C_t + C_s)$. $R_t = \sum_j p_{jt} y_{jt}$ is the total revenue and $R_m = .5(R_t + R_s)$, then expression (A1) becomes:

$$\begin{aligned}&\sum_{j=1}^{m} .5\left(p_{jt} + p_{js}\right)\frac{C_m}{R_m}(y_{jt} - y_{js}) - \sum_{i=1}^{n} .5(\omega_{it} + \omega_{is})(v_{it} - v_{is}) \\ &= \sum_{j=1}^{m} .5\left(p_{jt} + p_{js}\right)\frac{C_m}{R_m}(y_{jt} - y_{js}) - \sum_{j=1}^{m} .5\left(\frac{\partial C}{\partial y_{jt}} + \frac{\partial C}{\partial y_{js}}\right)(y_{jt} - y_{js}) \\ &\quad -.5\left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial s}\right)(t - s)\end{aligned} \tag{A2}$$

Letting $\sigma_{jt} = p_{jt} y_{jt} / R_t$, be the $j$the output revenue share, $\eta_{jt} = \left(\partial C / \partial y_{jt}\right)(y_{jt} / C_t)$ the cost elasticity with the respect to jth output, and $\xi_{vt} = -(\partial C / \partial t) / C_t$ the input-based rate of technological change, then (A2) can be rewritten as:

$$\begin{aligned}&\sum_{j=1}^{m} .5\left(p_{jt} + p_{js}\right)\frac{C_m}{R_m}(y_{jt} - y_{js}) - \sum_{i=1}^{n} .5(\omega_{it} + \omega_{is})(v_{it} - v_{is}) \\ &= \sum_{j=1}^{m} .5\left[\left(\sigma_{jt}\frac{R_t}{y_{jt}}\frac{C_m}{R_m} - \eta_{jt}\frac{C_t}{y_{jt}}\right) + \left(\sigma_{js}\frac{R_s}{y_{js}}\frac{C_m}{R_m} - \eta_{js}\frac{C_s}{y_{js}}\right)\right](y_{jt} - y_{js}) \\ &\quad +.5\left(\xi_{vt} C_t + \xi_{vs} C_s\right)(t - s)\end{aligned} \tag{A3}$$

Multiply and divide the first term of (A3) by y_j/R and the second term by v_i/C in the appropriate time period and collecting terms provides:

$$\begin{aligned}
&\sum_{j=1}^{m} .5\left(\frac{p_{jt}y_{jt}}{R_t}\frac{(y_{jt}-y_{js})R_t}{y_{jt}}\frac{C_m}{R_m} + \frac{p_{js}y_{js}}{R_s}\frac{(y_{jt}-y_{js})R_s}{y_{js}}\frac{C_m}{R_m}\right) \\
&-\sum_{i=1}^{n} .5\left(\frac{\omega_{it}v_{it}}{C_t}\frac{(v_{it}-v_{is})C_t}{v_{it}} + \frac{\omega_{is}v_{is}}{C_s}\frac{(v_{it}-v_{is})C_s}{v_{is}}\right) \\
&= \sum_{j=1}^{m} .5\left[\left(\sigma_{jt}\frac{R_t}{y_{jt}}\frac{C_m}{R_m} - \eta_{jt}\frac{C_t}{y_{jt}}\right) + \left(\sigma_{js}\frac{R_s}{y_{js}}\frac{C_m}{R_m} - \eta_{js}\frac{C_s}{y_{js}}\right)\right](y_{jt}-y_{js}) \\
&+.5(\xi_{vt}C_t + \xi_{vs}C_s)(t-s)
\end{aligned} \tag{A4}$$

Next multiply and divide the first and third terms of (A4) by $y_{jm} = .5(y_{jt}+y_{js})$ and the second term by $v_{im} = .5(v_{it}+v_{is})$ yields:

$$\begin{aligned}
&\sum_{j=1}^{m} .5\left[\sigma_{jt}\frac{(y_{jt}-y_{js})y_{jm}R_t}{y_{jm}y_{jt}}\frac{C_m}{R_m} + \sigma_{js}\frac{(y_{jt}-y_{js})y_{jm}R_s}{y_{jm}y_{js}}\frac{C_m}{R_m}\right] \\
&-\sum_{i=1}^{n} .5\left[\breve{s}_{it}\frac{(v_{it}-v_{is})v_{im}C_t}{v_{im}v_{it}} + \breve{s}_{is}\frac{(v_{it}-v_{is})v_{im}C_s}{v_{im}v_{is}}\right] \\
&= \sum_{j=1}^{m} .5\left[\left(\sigma_{jt}\frac{y_{jm}R_t}{y_{jt}}\frac{C_m}{R_m} - \eta_{jt}\frac{y_{jm}C_t}{y_{jt}}\right) + \left(\sigma_{js}\frac{y_{jm}R_s}{y_{js}}\frac{C_m}{R_m} - \eta_{js}\frac{y_{jm}C_s}{y_{js}}\right)\right]\frac{(y_{jt}-y_{js})}{y_{jm}} \\
&+.5(\xi_{vt}C_t + \xi_{vs}C_s)(t-s)
\end{aligned} \tag{A5}$$

Now TFP growth between periods, s and t embodying the margins associated with costly disinvestment is defined as adjusted $TFPG^a(s,t) = \dot{Y}/Y - \dot{\Psi}/\Psi,$ where

$$(\dot{Y}/Y) = \sum_{j=1}^{m} .5\left[\sigma_{jt}\frac{(y_{jt}-y_{js})}{y_{jm}}\frac{y_{jm}R_t}{R_m y_{jt}} + \sigma_{js}\frac{(y_{jt}-y_{js})}{y_{jm}}\frac{y_{jm}R_s}{R_m y_{js}}\right]$$

is the output quantity growth rate and

$$(\dot{\Psi}/\Psi) = \sum_{i=1}^{n} .5\left[\breve{s}_{it}\frac{(v_{it}-v_{is})}{v_{im}}\frac{v_{im}C_t}{C_m v_{it}} + \breve{s}_{is}\frac{(v_{it}-v_{is})}{v_{im}}\frac{v_{im}C_s}{C_m v_{is}}\right]$$

is the adjusted input quantity growth rate, such that the ith cost share is defined as $s_{it} = \omega_{it} v_{it} / C_t$ and so it includes the marginal commitment premia associated with disinvestment. With the definition of adjusted TFP growth, divide (A5) by C_m to obtain

$$TFPG^a(s,t) = \sum_{j=1}^{m} .5\left[\left(\sigma_{jt}\frac{R_t}{y_{jt}}\frac{C_m}{R_m} - \eta_{jt}\frac{C_t}{y_{jt}}\right)\frac{y_{jm}}{C_m} + \left(\sigma_{js}\frac{R_s}{y_{js}}\frac{C_m}{R_m} - \eta_{js}\frac{C_s}{y_{js}}\right)\frac{y_{jm}}{C_m}\right]\frac{(y_{jt} - y_{js})}{y_{jm}} + .5\left(\xi_{vt} C_t + \xi_{vs} C_s\right)\frac{1}{C_m}(t-s). \tag{A6}$$

Now adjusted TFP growth can be further simplified by recognizing that it is always possible to write:

$$p_{jt} = \psi_{jt}\frac{\partial C}{\partial y_{jt}},\ j = 1,...,m,$$

where $\psi_{jt} \geq 0$ is called the jth output margin in period t. This relationship does not imply profit maximization or any specific pricing rule for that matter. It just signifies for the jth output, a number $\psi_{jt} \geq 0$ can always be found, which in general differs across outputs and across time periods and equates price to marginal cost. Multiplying the above expression by y_{jt} and summing over j provides:

$$\sum_{j=1}^{m} p_{jt} y_{jt} = \sum_{j=1}^{m} \psi_{jt}\frac{\partial C}{\partial y_{jt}}\frac{y_{jt}}{C_t} C_t = \left(\sum_{j=1}^{m} \psi_{jt}\eta_{jt}\right) C_t$$

where the left side is the total revenue and the right hand side is the total cost multiplied by the sum of the product of output margins and output elasticities of cost. Thus, total cost is proportional to total revenue such that the proportionality factor is $\sum_{j=1}^{m} \psi_{jt}\eta_{jt}$.[18]

[18] The focus of the paper is not on output price-cost margins. Clearly, there is a relationship between the commitment premium for the user cost of capital and price-cost margins. In a single output context, the relationship is relatively straightforward but does require knowledge of the degree of returns to scale. This is readily seen from the right side of the previous expression. If there is a single output, or a common markup then:
$\left(\sum_{j=1}^{m} \psi_{jt}\eta_{jt}\right) C_t = \psi_t \eta_t \left(\sum_{i=1}^{n} \mu_{it} w_{it} v_{it}\right)$, where $\left(\sum_{j=1}^{m} \eta_{jt}\right) = \eta_t$ is the inverse of the degree of returns to scale, and ψ_t is the common or single markup. However, in a multiple output context the relationship between the multiple price-cost margins and commitment premium requires knowledge of the marginal costs of production, and thereby knowledge of the general cost function.

Now with $\kappa_t = \sum_{j=1}^{m} \psi_{jt} \eta_{jt}$, the previous expression can be written as

$$R_t = \kappa_t C_t = \breve{C}_t$$

where $p_{jt} = \partial \breve{C} / \partial y_{jt}$. From the definition of $\breve{C}_t$ it is possible to define an artificial input such that this $n+1$ st input's price is $\omega_{n+1t} = R_t - C_t = (\kappa_t - 1) C_t$, and its quantity is $v_{n+1t} = 1$, whereby $R_t = \breve{C}_t$, $= \sum_{i=1}^{n+1} \omega_{it} v_{it}$. With this accounting convention (A6) becomes

$$\begin{aligned} & TFPG_n^a(s,t) \\ & = \sum_{j=1}^{m} .5 \left[\left(\sigma_{jt} \frac{R_t}{y_{jt}} \frac{R_m}{R_m} - \breve{\eta}_{jt} \frac{R_t}{y_{jt}} \right) \frac{y_{jm}}{R_m} + \left(\sigma_{js} \frac{R_s}{y_{js}} \frac{R_m}{R_m} - \breve{\eta}_{js} \frac{R_s}{y_{js}} \right) \frac{y_{jm}}{R_m} \right] \frac{(y_{jt} - y_{js})}{y_{jm}} \\ & + .5 \left(\breve{\xi}_{\nu t} R_t + \breve{\xi}_{\nu s} R_s \right) \frac{1}{R_m} (t-s). \end{aligned} \qquad \text{(A7)}$$

where a $(\tilde{\ })$ refers to the output elasticities defined with respect to the cost $\tilde{C}$. $TFPG_n^a(s,t)$ is the adjusted TFP growth inclusive of the accounting convention so that

$$(\dot{\Psi} / \Psi) = \sum_{i=1}^{n+1} .5 \left[\breve{s}_{it} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im} R_t}{R_m v_{it}} + \breve{s}_{is} \frac{(v_{it} - v_{is})}{v_{im}} \frac{v_{im} R_s}{R_m v_{is}} \right]$$

is the new adjusted input growth rate, such that the ith cost share is defined as $\breve{s}_{it} = \omega_{it} v_{it} / \breve{C}_t = \omega_{it} v_{it} / R_t$. Notice for the $n+1$st input $(v_{it} - v_{is}) = 0$, as quantity is always unity. Since $p_{jk} = \partial \breve{C} / \partial y_{jk} \quad k = t, s$, then $\sigma_{jk} = \breve{\eta}_{jk}$ and so (A7) becomes:

$$TFPG_n^a(s,t) = .5 \left(\breve{\xi}_{\nu t} R_t + \breve{\xi}_{\nu s} R_s \right) \frac{1}{R_m} (t-s). \qquad \text{(A8)}$$

Expression (A8), which is equation (11) in the main body of the paper, shows that adjusted TFP growth consists of a technological change component where technological change is defined as $\breve{\xi}_{\nu t} = -(\partial \breve{C} / \partial t) / \breve{C}_t$.

Table 1: PARAMETER ESTIMATES

(Standard Error in Parenthesis)

Parameter	**Estimates**					
	Static Exp.		*AR Exp.*		*Instrumental Variable*	
μ_K	1.7832	1.7009	2.3815	1.6909	1.8267	1.7850
	(1.2327)	(0.9268)	(1.1844)	(0.8768)	(1.3523)	(0.8955)
ξ	0.0149	0.0493	0.0069	0.0497	0.0127	0.0572
	(0.0161)	(0.0145)	(0.0157)	(0.0152)	(0.0175)	(0.0191)
ξ_T		-0.0032		-0.0033		-0.0038
		(0.0010)		(0.0100)		(0.0014)
φ_K			0.3259	0.1069		
			(0.1801)	(0.1007)		
θ_K			0.7635	0.9180		
			(0.1272)	(0.0727)		
$S.E.R$	0.0316	0.0271	0.0286	0.0247	0.0332	0.0284

Table 2: USER COSTS AND RATES OF RETURN

Period	**User Cost** $\omega_{Kt} = \mu_K w_{Kt}$		**Rate of Return** ρ^*	
	Static Exp.	*AR Exp.*	*Static Exp.*	*AR Exp.*
1986	0.2661	0.2832	0.101	0.143
1987	0.2578	0.2693	0.196	0.140
1988	0.2715	0.2922	0.095	0.144
1989	0.2617	0.2759	0.149	0.141
1990	0.2648	0.2810	0.147	0.142
1991	0.2672	0.2851	0.126	0.143
1992	0.2650	0.2813	0.128	0.142
1993	0.2632	0.2784	0.118	0.142
1994	0.2591	0.2715	0.156	0.141
1995	0.2637	0.2792	0.138	0.142
1996	0.2641	0.2798	0.137	0.142
1997	0.2643	0.2802	0.095	0.142
1998	0.2548	0.2644	0.088	0.139
1999	0.2440	0.2463	0.128	0.136
2000	0.2424	0.2436	0.129	0.136
2001	0.2408	0.2410	0.125	0.135
2002	0.2386	0.2373	0.136	0.134
Mean	0.2582	0.2700	0.129	0.140
Std. Dev.	0.0100	0.0167	0.0255	0.0030

Table 3: TOTAL FACTOR PRODUCTIVITY GROWTH RATES

Period	**Observed TFPG** $TFPG^{o}$	**Norm. Obs. TFPG** $TFPG^{o}_{n}$	**Margin Component** $(\mu_K - 1) s_K \hat{v}_K$	**Tech. Change Component** $TFPG^{a}_{n}$	**Adjusted TFPG** $TFPG^{a}$
1986	0.0352	0.0351	0.0111	0.0431	0.0242
1987	0.0423	0.0423	0.0082	0.0398	0.0341
1988	-0.0013	0.0044	0.0107	0.0365	-0.0023
1989	0.0255	0.0284	0.0071	0.0331	0.0249
1990	0.0688	0.0680	0.0081	0.0298	0.0593
1991	0.0320	0.0332	0.0075	0.0265	0.0293
1992	0.0482	0.0477	0.0070	0.0232	0.0400
1993	0.0365	0.0368	0.0066	0.0199	0.0316
1994	0.0180	0.0199	0.0057	0.0166	0.0195
1995	0.0337	0.0340	0.0060	0.0133	0.0294
1996	0.0858	0.0838	0.0093	0.0100	0.0709
1997	0.0042	0.0076	0.0086	0.0067	0.0065
1998	0.0330	0.0337	0.0093	0.0034	0.0263
1999	0.0278	0.0296	0.0114	0.0001	0.0203
2000	-0.0234	-0.0192	0.0168	-0.0032	-0.0315
2001	0.0151	0.0128	0.0167	-0.0065	-0.0045
2002	-0.0279	-0.0290	0.0037	-0.0099	-0.0348
Mean	0.0267	0.0276	0.0090	0.0166	0.0202
Std. Dev	0.0288	0.0277	0.0035	0.0167	0.0275